STABILITY OF A SUPERSONIC BOUNDARY LAYER WITH RESPECT TO THREE-DIMENSIONAL DISTURBANCES

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The stability of a supersonic boundary layer over an intensively cooled plate with respect to three-dimensional disturbances is investigated. Two neutral stability curves, the existence of which was established in [1], are contemplated. It is shown by asymptotic analysis that each of these two neutral stability curves separates into a closed and an ordinary neutral curve in a certain range of disturbance propagation angles. As the surface is cooled, the closed neutral curve contracts to a point. The results of asymptotic analysis were confirmed by numerical integration of the stability equations.

1. The effect of three-dimensionality of disturbances on the stability of a supersonic boundary layer over a cooled surface was investigated in [2]. It was shown that reduction in the surface temperature increases the critical Reynolds number R^* of stability loss and reduces the instability region for all three-dimensional disturbances. However, in contrast to the case of two-dimensional disturbances [3], complete stabilization occurs only in the range of angles $0 < \chi < \chi^*$, where χ is the angle between the direction of disturbance propagation and the direction of the oncoming flow, and $\chi^* = \arccos M^{-1}$; M is the Mach number of the oncoming flow. For $\chi > \chi^*$ and at any surface temperature, there are neutral and increasing disturbances, but the values of R^* for them are so large (the Reynolds numbers along the thickness of the boundary layer are equal to $\sim 10^6$) that they apparently have no practical significance. The critical values of the surface temperature $T^* = T^*(\chi)$, below which complete stabilization of the boundary layer on a flat plate occurs, were calculated in [2] for $M = 4$ by using the asymptotic method (solid curve in Fig. 1a).

The complete stabilization temperature was determined from the condition that must be satisfied for $c \geq c_s$:

$$v(c, T) \geqslant \max [\psi_i(z)]. \tag{1.1}$$

Here, c is the phase velocity of the disturbance, T is the surface temperature, v(c, T) is the function of nonviscous solutions whose shape for $M = 4$ is shown in Fig. 1b, $\psi_i(z)$ is the imaginary part of the function of viscous solutions whose maximum as a function of c is marked by the dashed curve in Fig. 1b, and c_s is the lowest phase velocity at which solutions damped outside the boundary layer are possible.

We consider here the neutral curves at surface temperatures lower than the temperatures of complete nonviscous stabilization. For such neutral curves, $c \to c_s$ at both the lower and the upper asymptote. The value of c_s corresponds to the Reynolds number $R_s = \infty$ [3],

$$c_s = 1 - (M \cos \chi)^{-1}. \tag{1.2}$$

If, for a fixed value of T and for $c > c_s$, v(c) is a monotonic function of c, the complete stabilization temperature is determined from the condition [2, 3]

$$v(c_s, T) = \max [\psi_i(z)]. \tag{1.3}$$

Solution of Eq. (1.3) yields the dependence $T(\chi)$ which is shown in Fig. 1a by the curve hbdef. The segments hb and ef coincide with the conditions of complete stabilization. The significance of the dashed curve,

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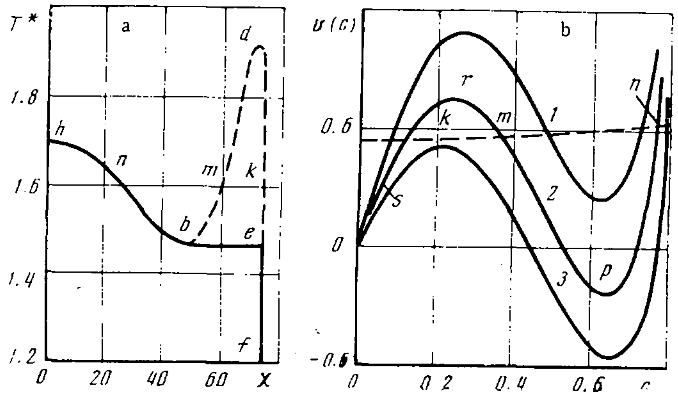

Fig. 1

which is a consequence of the nonmonotonic behavior of v(c), has not been explained in [2]. It was assumed there that at a certain surface temperature, the neutral curve can separate into two curves, one of which is stabilized in the usual manner, while the other persists for finite values of R.

As was mentioned in [2], confirmation of this assumption requires a method of solving the stability equations for supersonic flow for finite wave numbers α ($\alpha = 0$ for the complete stabilization considered in [2]). Such a method was not available to Dunn and Lin, and they did not proceed with their analysis any further.

Several methods for solving the stability equations numerically have been developed in recent years. This makes it possible to complete the asymptotic analysis [2] and verify the conclusions reached as a result of this analysis by accurate numerical calculations.

2. We shall continue the asymptotic analysis performed in [2] on the basis of Figs. 1a and b. Consider the behavior of the neutral curves for a fixed surface temperature. We choose the temperature T_1 at which max $[\psi_i(z)]$ intersects three times the dependence v(c) in Fig. 1b (curve 2) and the angle of disturbance propagation $\chi_1 \leqslant \chi^*$ (the angle for which c_s is close to zero).

Then, v(c) is a monotonic function of c along the segment sk (Fig. 1b), and an ordinary neutral curve must exist. Each c from $c_k > c > c_z$ corresponds to two values of $\psi_i(z)$, one for the upper branch of the neutral curve, and the other for the lower branch [3]. For $c = c_k$, the upper and the lower branches fuse, in which case the following condition is satisfied [3]:

$$v(c) = \max [\psi_i(z)] \tag{2.1}$$

Along the segment mpn (Fig. 1b), v(c) is a nonmonotonic function of c. For c from $c_m < c < c_n$, v(c) < max $[\psi_i(z)]$, and solutions corresponding to neutral and intensifying disturbances are possible. The function v(c) intersects $\psi_i(z)$ twice, i.e., there must be two branches of the neutral curve, which fuse for $c = c_m$ and $c = c_n$ [Eq. (2.1) is satisfied for these values of c], bounding a closed instability region. Thus, for the temperature T_1, there must be a closed neutral curve in addition to the usual neutral curve, determined by the segment sk of the v(c) dependence.

As the angle decreases in the $\chi < \chi_1$ range, the lower limit for c, determined by Eq. (1.2) increases. Only one closed neutral curve exists for angles for which $c_s > c_k$. This region is bounded by the dashed curve bdc in Fig. 1a.

At the angle for which $c_s = c_m$, the Reynolds number corresponding to this point of the neutral curve is equal to $R_m = R_s = \infty$, while for smaller angles the neutral curve is open. This is the curve bd in Fig. 1a.

Consider the behavior of the neutral curves as the surface temperature varies for a fixed angle χ_1.

The behavior of the neutral curve corresponding to the sk segment of the v(c) dependence with changes in the surface temperature is the same as the behavior of the neutral curve for two-dimensional disturbances. With a reduction in temperature for $T < T_1$, the points s and k of the v(c) dependence move closer to each other (curve 1 in Fig. 1b). As the temperatures marked by the curve def in Fig. 1a are reached, the Reynolds number R approaches infinity for all points of the neutral curve, i.e., complete stabilization sets in [2, 3].

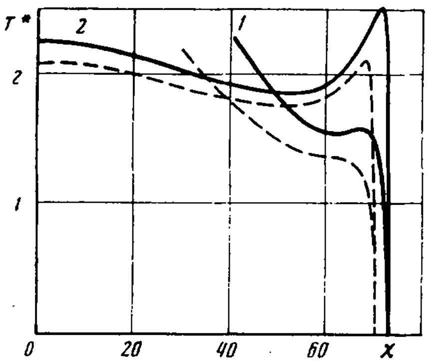

Fig. 2

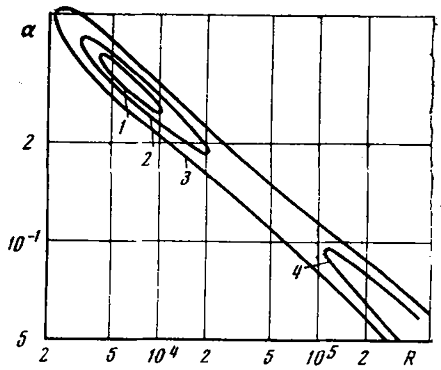

Fig. 3

The closed neutral curve corresponding to the mpn segment of the v(c) dependence behaves differently with temperature variations. With a reduction in the surface temperature, for $T < T_1$, the points m and n draw nearer to each other (Fig. 1b), and, at the temperature for which the point p touches the max $[\psi_i(z)]$ curve, the closed neutral curve contracts to a point. This temperature is determined by the tangent at the point b in Fig. 1a. Since the condition $c > c_s$ is satisfied for all value of c in the interval $c_m < c < c_n$, contraction of the closed neutral curve occurs for finite values of R. For $T > T_1$, the points k and m move closer to each other, and, at the temperature for which the point r touches the max $[\psi_i(z)]$ curve, the closed neutral curve fuses with the ordinary neutral curve. This temperature is determined by the tangent at the point d in Fig. 1a.

We have considered the neutral curve determined by the first maximum of $\psi_i(z)$ for $z \sim 3$ (dashed curve in Fig. 1a). Besides this neutral curve, there is another viscous neutral curve at low surface temperatures [1], which is determined by the second maximum of $\psi_i(z)$. For small values of c, the modified Tietjens function $F_i(z)$ provides a very good approximation of $\psi_i(z)$. Then, the second neutral curve is determined by the second maximum of $F_i(z)$, which manifests itself at $z \sim 6$. The values of F(z) up to $z = 10$ are given in [4]. The above considerations hold for this curve; however, since the first maximum of $\psi_i(z)$ (~ 0.05) is larger than the second by one order of magnitude, the temperatures at which a closed neutral curve develops and contracts to a point will be higher than those for the neutral curve considered earlier.

3. The results of asymptotic analysis were checked by numerical calculations. The investigation was performed on the basis of numerical integration of the Lees and Lin stability equations, supplemented by terms comprising the transverse velocity of the basic flow [5]. These equations hold for two-dimensional disturbances. Three-dimensionality was taken into account by using the approximate method of Dunn and Lin [2]. The following substitution was made in the stability equations for two-dimensional disturbances:

$$M^\circ = M \cos\chi, \quad R^\circ = R\cos\chi, \quad \alpha^\circ R^\circ = \alpha R, \quad c^\circ = c.$$

The results obtained by means of the method of Dunn and Lin for small supersonic Mach numbers ($M < 3$) are close to the results obtained by accurate accounting for the disturbance three-dimensionality [6]. However, this method reduces considerably the computer time. The complete stabilization temperatures were determined by using the numerical method proposed in [1]. The neutral stability curves were calculated by means of a method [7] whereby the neutral curve can be plotted on the basis of its complete stabilization conditions.

The mean values to be substituted in the stability equations were calculated for a boundary layer on a flat plate for $\gamma = 1.41$, the Prandtl number $\sigma = 0.72$, $T_\infty = 157°$, and the viscosity obeying Sutherland's law. The method of solution has been described in [8].

In the first series of calculations, we determined the complete stabilization temperatures for two "viscous" neutral curves for $M = 3.5$ (the existence of these curves for two-dimensional disturbances has been demonstrated in [1] and investigated in [7]). The calculation results are given in Fig. 2. The temperatures of complete stabilization for the first curve [corresponding to the first maximum of $\psi_i(z)$], are marked by 1, while the complete stabilization temperatures for the second curve [corresponding to the second maximum of $\psi_i(z)$] are marked by 2. For $\chi = 40°$, the first neutral curve vanishes as it did for a certain number M for two-dimensional disturbances [1, 7]. Only the second neutral curve exists for $\chi < 40°$ and $M = 3.5$.

A peculiarity of the $T^*(\chi)$ dependence for both the first and the second neutral curve is its nonmono-

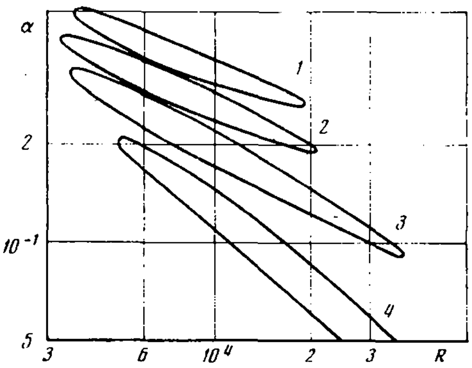

Fig. 4

tonic behavior for the angle ~65°. This can be explained by the fact that determination of complete stabilization temperatures by means of the numerical method [1] is equivalent to the solution of Eq. (1.3), i.e., the results obtained by means of the numerical method are equivalent to the curve hbdf in Fig. 1a. Then, according to asymptotic analysis, there must be closed neutral curves which contract to a point with a reduction in T.

The nonmonotonic behavior of $T^*(\chi)$ does not occur for every number M. The dashed curve in Fig. 2 indicates the complete stabilization temperatures for M = 3. For the first neutral curve, $T^*(\chi)$ is a monotonic function; the closed neutral curve does not develop. For the second neutral curve, the existence domain of the closed neutral curve is reduced considerably. A similar smoothing-out of the $T^*(\chi)$ dependence with a reduction in M was observed in asymptotic determination of complete stabilization temperatures [9]. The behavior of the neutral curve for $\chi_1 = 71°$ with variation of the surface temperature T was investigated in the second series of calculations. The calculation results are given in Fig. 3.

A single neutral curve exists for T = 1.6 (curve 3 in Fig. 3). With a reduction in the surface temperature, it divides, so that two neutral curves exist at T = 1.562 (2 and 4), one of which is closed (2). The critical Reynolds number R* for the open neutral curve is larger than R* for the closed curve by two orders of magnitude. With a reduction in the surface temperature, the value of R* for this neutral curve increases even more, and, therefore, it is not considered any further. For $T_1 = 1.551$, the instability region diminishes (curve 1 in Fig. 3) and contracts to a point.

The behavior of the closed neutral curve for $T_1 = 1.562$ with changes in χ (Fig. 4) was investigated in the third series of calculations. For $\chi = 62.5°$ and the temperature $T > T^*$ in Fig. 4, the curve (curve 4) is open. As the angle increases, the neutral curve closes, and only the closed curve (3) exists for $\chi = 67°$. The curve remains closed with a further increase in χ. The minimum value of R* occurs for $\chi = 71°$ (for a thermally insulated surface, the most critical disturbances are those which propagate at the angle $\chi \sim 60°$).

Thus, the basic conclusions of asymptotic analysis concerning the behavior of neutral curve 1 with variations of T and χ have been confirmed. However, calculations show that the temperature at which the closed neutral curve vanishes lies somewhat lower than the tangent at the point b.

The behavior of neutral curve 2 (Fig. 2) with changes in χ and T is similar to the behavior of curve 1. However, the value of R* for this curve in the critical angle range ($\chi \sim 60/70°$) is much larger than for the first neutral curve, plotted for the same conditions.

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